# Constraints of $B_d \to \phi K_S$ on Fundamental Physics

Haibin Wang

Purdue University

with Gordon Kane, Liantao Wang, Ting T Wang

- I. Overview
- II. $B_d \to \phi K_S$, $B \to X_s\gamma$ and EDM: weak scale correlation
- III. Generic features at high scale
- IV. Numerical study
- V. Conclusion

# I. Overview

- Experimental results

| Observable | BaBar | Belle |
|---|---|---|
| $Br$ (in $10^{-6}$) | $8.1\,^{+3.1}_{-2.5} \pm 0.8$ | $8.7\,^{+3.8}_{-3.0} \pm 1.5$ |
| $S_{\phi K_S}$ | $0.45 \pm 0.43 \pm 0.07$ | $-0.96 \pm 0.50\,^{+0.09}_{-0.11}$ |
| $C_{\phi K_S}$ | $-0.80 \pm 0.38 \pm 0.12$ | $0.56 \pm 0.41 \pm 0.16$ |

SM: $S_{\phi K} = 0.736 \pm 0.049$

- Why $B \to \phi K$ interesting?
  - A possible deviation from SM prediction
  - Information on the flavor structure

– The deviation, if ture, suggests one large FC parameter at high scale

MSSM: Large off-diagonal element/splitting in squark mass matrix

– Propagate in RGE running and induce more FC

– Clues/guidance for underlying theory

– Even if not confirmed, provide a paradigm to study interplay between FC processes

- CP asymmetry of $B_d \to \phi K_S$

$$\mathcal{A}_{\phi K}(t) \equiv \frac{\Gamma(\bar{B}^0_{\rm phys}(t) \to \phi K_S) - \Gamma(B^0_{\rm phys}(t) \to \phi K_S)}{\Gamma(\bar{B}^0_{\rm phys}(t) \to \phi K_S) + \Gamma(B^0_{\rm phys}(t) \to \phi K_S)}$$

$$= -C_{\phi K}\cos(\Delta M t) + S_{\phi K}\sin(\Delta M t),$$

$$S_{\phi K} = \frac{2\,{\rm Im}\lambda_{\phi K}}{1+|\lambda_{\phi K}|^2},$$

$$\lambda_{\phi K} \equiv -e^{-2i(\beta+\theta_d)} \frac{\bar{A}(\bar{B}^0 \to \phi K_S)}{A(B \to \phi K_S)}.$$

- $S_{\phi K}$ implies a new phase!

$$\begin{aligned}
\lambda_{\phi K_S} &= -\frac{V_{td}V_{tb}^*}{V_{td}^*V_{tb}} \frac{V_{ts}^*V_{tb}(1+\frac{b}{a}\frac{\delta}{V_{ts}^*V_{tb}})}{V_{ts}V_{tb}^*(1+\frac{b}{a}\frac{\delta^*}{V_{ts}V_{tb}^*})} \frac{V_{cs}V_{cd}^*}{V_{cs}^*V_{cd}} \\
&= -e^{-i2\beta} \frac{1+re^{i\theta}}{1+re^{-i\theta}}
\end{aligned}$$

  – $\delta$ a complex parameter of new physics

  – Rephasing invariant since $\lambda_{\phi K_S}$ is physical

  – MSSM: use $\delta_{23}^{d,LR}$, the invariant

$$\frac{\delta_{23}^{d,LR}}{V_{ts}^*V_{tb}} \frac{M_3}{|M_3|}$$

# II. $B_d \to \phi K_S$, $B \to X_s\gamma$ and Mercury EDM: weak scale interplay

- Observables

  – $S_{\phi K_S}$

  Gluino contribution

$$C_{8g} \; : \; (\delta^d_{LR})_{23}, \;\; (\delta^d_{LL})_{23}(\delta^d_{LR})_{33}$$

$$C'_{8g} \; : \; (\delta^d_{LR})_{32}, \;\; (\delta^d_{RR})_{32}(\delta^d_{RL})_{33}$$

$$|(\delta^d_{LR})_{23}| \;\sim\; 0.01$$

  Chargino contribution

  – Mercury EDM: CEDM of strange quark $\tilde{d}_C$

$$e|\tilde{d}^C| < n \cdot 5.5 \times 10^{-25} \; ecm$$

  ∗ Gluino contribution

$$\mathrm{Im}((\delta^d_{LR})_{22}, \;\; (\delta^d_{LL})_{23}(\delta^d_{LR})_{32}, \;\; (\delta^d_{LL})_{23}(\delta^d_{LR})_{33}(\delta^d_{RR})_{32})$$

$*$ Chargino contribution

$$\mathrm{Im}(V_{ts}(\delta^u_{LR})_{23}, \quad V_{ts}(\delta^u_{LL})_{32}(\delta^u_{RL})_{33})$$

M. Ento, M. Kakizaki, M. Yamaguchi, hep-ph/0311072

$*$ double and triple mass insertion use $(\delta^{u,d}_{LL})_{23}$ naturally $O(0.01)$

– $Br(B \to X_s \gamma)$

$$2.0 \times 10^{-4} < Br(B \to X_s \gamma) < 4.5 \times 10^{-4}$$

$*$ Gluino contribution does not strongly constrain $S_{\phi K}$

G. Kane, P. Ko, H.b. Wang, C. Kolda, J.h. Park, L.T. Wang, hep-ph/0212092

$*$ Chargino contributions

$$(\delta^u_{LL})_{23}$$

- The correlation

| $S_{\phi K_S}$ | $Br(B \to X_s\gamma)$ | CEDM |
|---|---|---|
| $(\delta^d_{LR})_{23}$ | $\cdot$ | $\cdot$ |
| $(\delta^d_{LL})_{23}(\delta^d_{LR})_{33}$ | $(\delta^u_{LL})_{23}$ | $V_{ts}(\delta^u_{LL})_{32}(\delta^u_{RL})_{33}$ |
| $(\delta^d_{LR})_{32}$ | $\cdot$ | $(\delta^d_{LL})_{23}(\delta^d_{LR})_{32}$ |
| $(\delta^d_{RR})_{32}(\delta^d_{RL})_{33}$ | $\cdot$ | $(\delta^d_{LL})_{23}(\delta^d_{LR})_{33}(\delta^d_{RR})_{32}$ |

- $|(\delta^{u,d}_{LL})_{23}| \sim O(0.01)$ In SCKM basis

$$(\delta^{u,d}_{LL})_{23} \sim \eta y_t^2 \lambda^2_{CKM} \frac{(\tilde{m}^2_Q)_{33} + (\tilde{m}^2_Q)_{22} + 2(\tilde{m}^2_U)_{33} + 2(\tilde{m}^2_{H_u})}{\tilde{m}^2_q}$$

$$\sim 0.01$$

$$\eta = \frac{|t_W - t_\Lambda|}{16\pi^2} \sim 0.2$$

- $|(\delta^u_{RL})_{33}| = \eta \frac{m_t}{\tilde{m}_q} \frac{\tilde{A}^{SCKM}_{33}}{\tilde{m}_q} \sim O(0.1)$
- Fill in the table with generic features of fundamental physics
- Must use the full mass matrices for EDM and $B \to X_s\gamma$

# III. Generic features of boundary conditions at high scale

- Bi-unitary transformation of Yukawas, trilinears and soft masses
  - Basis rotation
  - Equivalence relation

$$
\begin{aligned}
Y'_f &= K_L \cdot Y_f \cdot K_R^{f\dagger}, \quad f = u, d \\
\tilde{A}'_f &= K_L \cdot \tilde{A}_f \cdot K_R^{f\dagger} \\
m_f^{2\prime} &= K^f m_f^2 K^{f\dagger} \\
m_Q^{2\prime} &= L_L m_Q^2 K_L^{\dagger}
\end{aligned}
$$

- Hermitian A term to suppress EDM
  - Neutron and Mercury EDM

$$
|\mathrm{Im}(\delta^d_{LR})_{11}| < 6.7 \times 10^{-8}
$$

$$|\mathrm{Im}(\delta^u_{LR})_{11}| < 6.7 \times 10^{-8}$$

$$|\mathrm{Im}(\delta^d_{LR})_{22}| < 5.6 \times 10^{-6}$$

S. Abel, A. Khalil and Lebedev, hep-ph/0103320

– $\mathrm{Im}\delta^{LR}_{ii}|_\Lambda = 0$

$$\begin{aligned} \mathrm{Im}\delta^{LR}_{22} &= \frac{m_s \mathrm{Im}\tilde{A}^{SCKM}_{22}}{\tilde{m}_q^2} \\ &\sim 0.2\frac{0.1 \cdot 200}{7 \cdot 200^2} = 1.4 \times 10^{-5} \end{aligned}$$

– Hermitian $A \rightarrow \mathrm{Im}\delta^{LR}_{ii}|_\Lambda = 0$

$$\begin{aligned} \tilde{A} &= A \cdot Y \\ \tilde{A}^{SCKM} &= (K_L A K_L^\dagger)(K_L Y K_R^\dagger) \end{aligned}$$

– RGE will not induce phase to $\delta^{LR}_{ii}$, with hermitian A

– Hermitian $A$ equivalent to real diagonal $A$ with corresponding $Y$

– Diagonal $A$ is well-motivated

$$\tilde{A}^f = m_{3/2}(A_L^f \cdot Y^f + Y^f \cdot A_R^f), \qquad f = U, D,$$

Yukawas are not functions of fields with non-vanishing $F$ term.

$$\delta_{23}^{d,LR} \approx \frac{m_b(A_{L3}^D - A_{L2}^D)(K_L^{D*})_{33}(K_L^D)_{23}}{m_{\tilde{q}}^2} \qquad (1)$$

T. Kobayashi, O. Vives, hep-ph/0011200

- Flavor dependent parameters

$$(\delta_{LL})_{23}, \quad (\delta_{RR})_{23},$$

$$(\delta_{LR})_{23}, \quad (\delta_{LR})_{32}, \quad (\delta_{LR})_{33}$$

– $|(\delta_{LR}^u)_{33}| \sim 0.1$

– $|(\delta_{LR}^d)_{33}| \simeq \frac{m_b \mu \tan\beta}{\tilde{m}_q^2} \sim 0.1$

– Ubiquitous $(\delta_{LR}^d)_{23}$

$*$ Induce $(\delta^d_{LL})_{23}$ to enhance $Br(B \to X_s\gamma)$

$$\begin{aligned}
(\delta^d_{LL})_{23} &\sim \eta \frac{2\tilde{A}^d_{23}\tilde{A}^d_{33}}{\tilde{m}^2_q} \\
&\sim 2\eta(\delta^d_{LR})_{23}\frac{\tilde{A}^d_{33}}{v}\tan\beta \\
&\sim \tan\beta(\delta^d_{LR})_{23} \sim O(0.1)
\end{aligned}$$

$*$ Enhance CEDM, if $A^L_u = A^L_d$

$$(\delta^u_{LR})_{23} = \frac{m_t}{m_b}(\delta^d_{LR})_{23}$$

- Fill the table, continued

| $S_{\phi K_S}$ | $Br(B \to X_s\gamma)$ | CEDM |
|---|---|---|
| $(\delta^d_{LR})_{23}$ | $(\delta^u_{LL})_{23}$ | $V_{ts}(\delta^u_{LR})_{23}$ |
| $(\delta^d_{LL})_{23}(\delta^d_{LR})_{33}$ | $(\delta^u_{LL})_{23}$ | $V_{ts}(\delta^u_{LL})_{32}(\delta^u_{RL})_{33}$ |
| $(\delta^d_{LR})_{32}$ | $\cdot$ | $(\delta^d_{LL})_{23}(\delta^d_{LR})_{32}$ |
| $(\delta^d_{RR})_{32}(\delta^d_{RL})_{33}$ | $\cdot$ | $(\delta^d_{LL})_{23}(\delta^d_{LR})_{33}(\delta^d_{RR})_{32}$ |

Blue due to RGE and/or high scale assumptions

- Loophole: Used weak/high scale value to evaluate integral

# IV. Numerical study

- Nearly-universal boundary conditions

  – Boundary conditions

$$
\begin{aligned}
\tilde{A}^f_{ij} &= m_{3/2}(A^f_L \cdot Y^f + Y^f \cdot A^f_R), \qquad f = U, D, \\
\tilde{m}^2_F &= m_0^2 \cdot Diag((m^2_F)_1, (m^2_F)_2, (m^2_F)_3), \\
M_1,\ M_2,\ M_3 &= m_{1/2}, \\
m^2_{H_u},\ m^2_{H_d} &= m_0^2. \\
\tan\beta &= 16
\end{aligned}
$$

  – Five scenarios, characterized by non-zero MI at high scale

$$
\begin{aligned}
(\delta_{LL})_{23} : &\qquad (\tilde{m}^2_Q)_{22} - (\tilde{m}^2_Q)_{33} \\
(\delta^{u,d}_{LR})_{23}\ (A^L_u = A^L_d) : &\qquad (A^{u,d}_L)_2 - (A^{u,d}_L)_3
\end{aligned}
$$

$$(\delta^d_{LR})_{23}\ (A^L_u \neq A^L_d): \qquad (A^d_L)_2 - (A^d_L)_3$$

$$(\delta^d_{LR})_{32}: \qquad (A^d_R)_2 - (A^d_R)_3$$

$$(\delta^d_{RR})_{23}: \qquad (\tilde{m}^2_d)_{22} - (\tilde{m}^2_d)_{33}$$

– $m_{h^0} = 114.4\ GeV \to m_{3/2} \sim 300\ GeV$

– Other flavor constraints satisfied

– Table again (Fig. 1)

- Tuning for $\delta^{LL}_{23}$ scenario (Fig. 2)
- Serve as a reference on how to avoid EDM and $B \to X_s\gamma$ constraints

| | $(\delta_{LL})_{23}$ | $(\delta_{LR})_{23}$<br>$(A_u = A_d)$ | $(\delta_{LR})_{23}$<br>$(A_u \neq A_d)$ | $(\delta^d_{LR})_{32}$ | $(\delta^d_{RR})_{23}$ |
|---|---|---|---|---|---|
| $S_{\phi K}$ | 0.13 | $-0.052$ | $-0.50$ | 0.55 (0.16) | 0.54 (0.21) |
| $Br(B \to X_s\gamma) \times 10^4$ | 4.5 | 4.0 | 4.5 | 2.2 (2.5) | 2.2 (2.5) |
| CEDM(3×CEDM)<br>$\times 10^{25} e$cm | 0.58 | 5.5 | 2.6 | 5.5 (16.5) | 5.5 (16.5) |

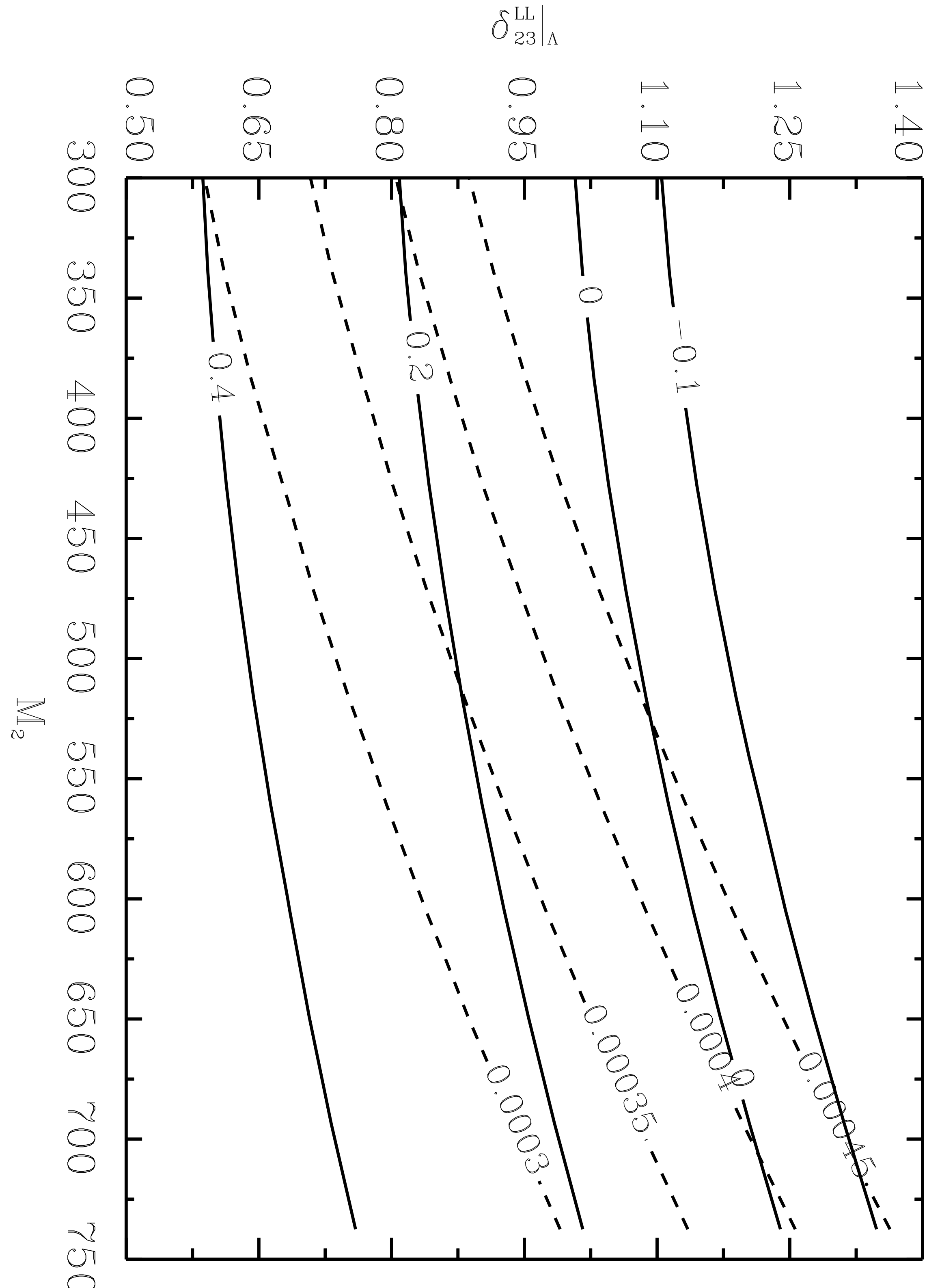




Figure 2:

# IV. Conclusion

- Beyond-SM $S_{\phi K}$ implies a new phase at weak scale
- Strong correlation among $B_d \to \phi K_S$, $B \to X_s \gamma$ and Mercury EDM
- A "large" high scale mass insertion feeds into other MIs
- The correlation becomes even more interesting
- Loopholes in our argument provide reference for high scale models
  - More than one MI at high scale, cancellations
  - A parameter change sign in running, reduce MI at low scale
  - There are other parameters