The Puzzle of Charge and Mass

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Outline

- Standard Model $\Longrightarrow$ SUSY GUTs
- Gauge coupling unification and proton decay
- Yukawa coupling unification for third family
- Hierarchy of fermion masses and mixing

  Family symmetry

Standard Model : Symmetry and Charges

Forces SPIN 1 Bosons

SYMMETRY $SU(3)_{QCD} \times [SU(2) \times U(1)_Y]$

Strong Electroweak

Couplings $\alpha_3 \quad > \quad \alpha_2 \quad > \quad \alpha_1$

	gluons	Weak bosons	photon
	8 **g**	$W^{\pm}, \quad Z^0$	γ

Matter SPIN 1/2 Fermions

Charges	$q = \begin{pmatrix} u \\ d \end{pmatrix}$	$\bar{u}$	$\bar{d}$	$l = \begin{pmatrix} \nu_e \\ e \end{pmatrix}$	$\bar{e}$
Y	$\frac{1}{3}$	$-\frac{4}{3}$	$\frac{2}{3}$	-1	$+2$

$Q_{EM} = T_3 + \frac{Y}{2}$

Masses and Mixing Angles

15 parameters of SM (not including ν s)

Mass

ν_e	**e**	**u**	**d**
$\leq 10^{-7}$	1/2	2	5
ν_μ	μ	**c**	**s**
$\leq 10^{-7}$	105.6	1,300	120
ν_τ	τ	**t**	**b**
$\leq 10^{-7}$	1,777	174,000	4,500

$W^{\pm}$ 80,000 Z^0 91,000 γ, gluon 0 Higgs ???

Mixing Angles

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

$$\approx \begin{pmatrix} 1 & \lambda & (\rho + i\eta)\lambda^3 \\ -\lambda & 1 & A\lambda^2 \\ (1-\rho+i\eta)A\lambda^3 & \lambda^2 & 1 \end{pmatrix}$$

Grand Unification & Charge Quantization

— Pati & Salam; Georgi & Glashow; Georgi; Fritzsch & Minkowski

Lepton Number is the Fourth Color !!

$$\mathbf{G}_{\text{Pati-Salam}} \equiv \quad \mathbf{SU_4}(\mathbf{color}) \times \mathbf{SU_2}(\mathbf{L}) \times \mathbf{SU_2}(\mathbf{R})$$

$$\psi = \begin{pmatrix} u & \nu_e \\ d & e \end{pmatrix} \qquad \psi^c = \begin{pmatrix} \bar{u} & \bar{\nu}_e \\ \bar{d} & \bar{e} \end{pmatrix} \qquad \mathcal{H} = (H_u \;\; H_d)$$

SU$_5$

$$\{Q = \begin{pmatrix} u \\ d \end{pmatrix} \quad \bar{e} \quad \bar{u}\} \quad \subset \quad \mathbf{10}$$

$$\{\bar{d} \quad L = \begin{pmatrix} \nu \\ e \end{pmatrix}\} \quad \subset \quad \bar{\mathbf{5}}$$

$$\begin{pmatrix} H_u \\ T \end{pmatrix}, \quad \begin{pmatrix} H_d \\ \bar{T} \end{pmatrix} \quad \subset \quad \mathbf{5_H}, \quad \bar{\mathbf{5}}_\mathbf{H}$$

SO$_{10}$

$$\mathbf{10} + \bar{\mathbf{5}} + \bar{\nu} \quad \subset \quad \mathbf{16}$$

$$\mathbf{5_H}, \quad \bar{\mathbf{5}}_\mathbf{H} \quad \subset \quad \mathbf{10_H}$$

Grand Unification – SO(10)

State	Y $= \frac{2}{3}\Sigma(\mathrm{C}) - \Sigma(\mathrm{W})$	Color C spins	Weak W spins
$\bar{\nu}$	0	$+\ +\ +$	$+\,+$
$\bar{\mathbf{e}}$	2	$+\ +\ +$	$-\,-$
$\mathbf{u_r}$	$\frac{1}{3}$	$-\ +\ +$	$+\,-$
$\mathbf{d_r}$		$-\ +\ +$	$-\,+$
$\mathbf{u_b}$		$+\ -\ +$	$+\,-$
$\mathbf{d_b}$		$+\ -\ +$	$-\,+$
$\mathbf{u_y}$		$+\ +\ -$	$+\,-$
$\mathbf{d_y}$		$+\ +\ -$	$-\,+$
$\bar{\mathbf{u}}_\mathbf{r}$	$-\frac{4}{3}$	$+\ -\ -$	$+\,+$
$\bar{\mathbf{u}}_\mathbf{b}$		$-\ +\ -$	$+\,+$
$\bar{\mathbf{u}}_\mathbf{y}$		$-\ -\ +$	$+\,+$
$\bar{\mathbf{d}}_\mathbf{r}$	$\frac{2}{3}$	$+\ -\ -$	$-\,-$
$\bar{\mathbf{d}}_\mathbf{b}$		$-\ +\ -$	$-\,-$
$\bar{\mathbf{d}}_\mathbf{y}$		$-\ -\ +$	$-\,-$
ν	-1	$-\ -\ -$	$+\,-$
$\mathbf{e}$		$-\ -\ -$	$-\,+$

Supersymmetry

Bosons	*SPIN 0, 1, 2*	*Fermions*	*SPIN 1/2, 3/2*
Bose-Einstein statistics		*Fermi-Dirac statistics*	
FORCES		*MATTER*	

Bosons $(\phi(t, \mathbf{x}))$ $\Leftrightarrow$ Fermions $(\psi(t, \mathbf{x}))$

Space-Time "rotation" in

SUPERSPACE : $\mathbf{z} = \{\ t, \mathbf{x}, \theta\ \}$

$[\theta, \theta]_+ = 0$ anti-commuting coordinates !!

- Superfield

$$\Phi(\ \mathbf{z}\ ) = \phi(t, \mathbf{x}) + \theta\ \psi(t, \mathbf{x})$$

- Local Supersymmetry $\Longrightarrow$ Supergravity
- Stabilizes light Higgs $\Longrightarrow$ no Gauge Hierarchy Problem !

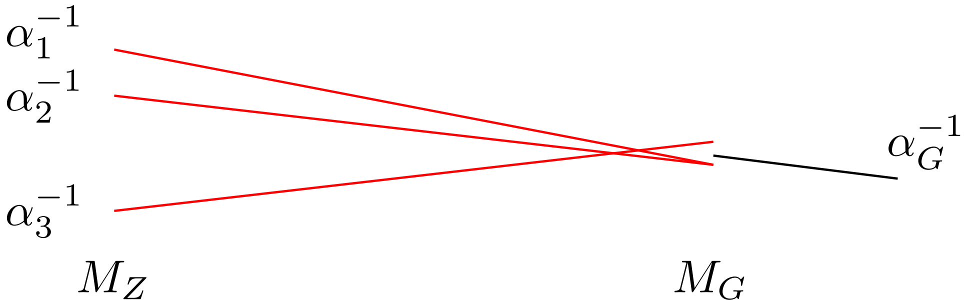
SUSY GUTs

- $M_Z << M_{GUT}$ "Natural"
- Large $M_{top} \Leftrightarrow m_h^2 < 0$ "Natural"
- Explains Charge Quantization
- Predicts Gauge Coupling Unification*
- Predicts Yukawa Coupling Unification
- + Family Symmetry $\Longrightarrow$ Hierarchy of Fermion Masses
- Neutrino Masses via See - Saw scale $\sim 10^{-3} - 10^{-2}\ M_G$
- LSP – Dark Matter Candidate
- Baryogenesis via Leptogenesis
- SUSY Desert $\Longrightarrow$ LHC experiments probe physics O(M_{Planck}) scale

Gauge coupling unification *

— Dimopoulos, S.R. & Wilczek; Dimopoulos & Georgi

* Only evidence for SUSY



- Significant GUT threshold corrections from Higgs and GUT breaking sectors

Def: $M_G \quad \Longleftrightarrow \quad \alpha_1(M_G) = \alpha_2(M_G) \equiv \tilde{\alpha}_G$

Using two loop RGE from M_Z to M_G, find

$$\begin{aligned} M_G &\approx \quad 3 \times 10^{16} \;\; \text{GeV} \\ \alpha_G^{-1} &\approx \qquad 24 \end{aligned}$$

Good fit requires:

$$\boxed{\epsilon_3 \equiv \frac{(\alpha_3(M_G) - \tilde{\alpha}_G)}{\tilde{\alpha}_G} \sim\text{-3\% to -4\%}}$$

Proton Decay and G.C. Unification

- V. Lucas and S. Raby, "Nucleon decay in a realistic SO(10) SUSY GUT," [arXiv:hep-ph/9610293].
- Minimal SUSY SU(5)

  T. Goto and T. Nihei, "Effect of RRRR dimension 5 operator on the proton decay in the minimal SU(5) SUGRA GUT model," [arXiv:hep-ph/9808255].

  K. S. Babu and M. J. Strassler, "A reexamination of proton decay in supersymmetric grand unified theories," [arXiv:hep-ph/9808447].

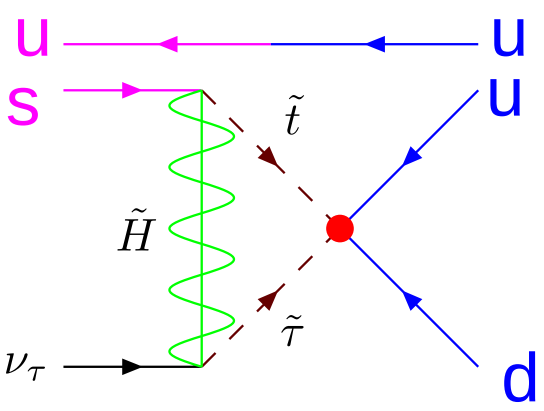
  H. Murayama and A. Pierce, "Not even decoupling can save minimal supersymmetric SU(5)," [arXiv:hep-ph/0108104].

  B. Bajc, P. F. Perez and G. Senjanovic, "Proton decay in minimal supersymmetric SU(5)," [arXiv:hep-ph/0204311].

- K. S. Babu, J. C. Pati and F. Wilczek, "Fermion masses, neutrino oscillations, and proton decay in the light of SuperKamiokande," [arXiv:hep-ph/9812538].
- G. Altarelli, F. Feruglio and I. Masina, "From minimal to realistic supersymmetric SU(5) grand unification," [arXiv:hep-ph/0007254].
- R. Dermisek, A. Mafi and S. Raby, "SUSY GUTs under siege: Proton decay," [arXiv:hep-ph/0007213].

Dimension 5 operators and proton decay

$$p \longrightarrow K^+ + \bar{\nu}_\tau$$



$$\text{Loop Factor} = \frac{\lambda_t\ \lambda_\tau}{16\pi^2}\,\frac{\sqrt{\mu^2+M_{1/2}^2}}{m_{16}^2}$$

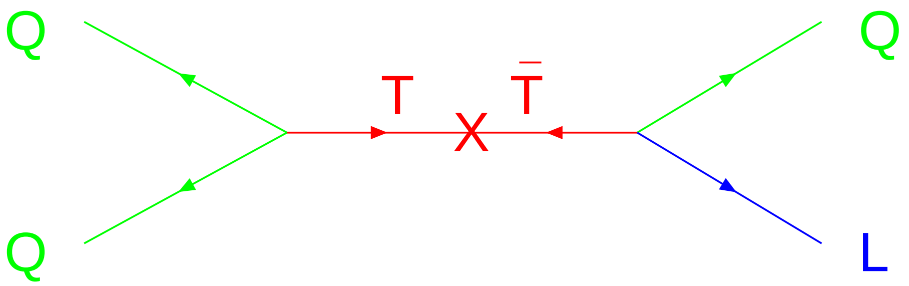
- Amplitude$(p \to K^+\bar{\nu}) \propto \frac{c\ c}{M_T^{eff}}\ (LoopFactor)$
- Loop Factor MINIMIZED

  $\mu, M_{1/2}$ SMALL; m_{16} Large
- M_T^{eff} MAXIMIZED $\Longrightarrow$

  Theory: $\tau_{p\to K^+\bar{\nu}} \leq (\frac{1}{3} - 3) \times 10^{34}$ yrs.

  Exp't: $\tau_{p\to K^+\bar{\nu}} > 2.2 \times 10^{33}$ yrs. – Super-K

Dimension 5 operator and Color Triplet Higgs mass



$$\frac{1}{M_T^{eff}}\left[\, Q\tfrac{1}{2}c_{qq}Q\ Qc_{ql}L+\overline{U}c_{ud}\overline{D}\ \overline{U}c_{ue}\overline{E}\,\right]$$

M_T^{eff} can be much greater than M_G

- $\frac{1}{M_T^{eff}} = (M_T^{-1})_{11}$

M_T : Higgs color triplet mass matrix

Example: $M_T = \begin{pmatrix} 0 & M_G \\ M_G & X \end{pmatrix} \Longrightarrow \frac{1}{M_T^{eff}} \equiv \frac{X}{M_G^2}$

- $X << M_G \Longrightarrow M_T^{eff} >> M_G$
- NO particle with mass greater than M_G
- Suppresses proton decay via dimension 5 ops.

M_T^{eff} and Gauge Coupling Unification

Recall : $\epsilon_3 \equiv \frac{(\alpha_3(M_G)-\tilde{\alpha}_G)}{\tilde{\alpha}_G} \sim -4\%$

$\epsilon_3 = \epsilon_3^{\rm Higgs} + \epsilon_3^{\rm GUT\ breaking} + \cdots$

$$\boxed{\epsilon_3^{\rm Higgs} = \frac{3\alpha_G}{5\pi} \ln(\frac{M_T^{eff}}{M_G})}$$

Model	Minimal SU_5	SU_5 "Natural" D/T	Minimal SO_{10}
$\epsilon_3^{\rm GUTbreaking}$ $\epsilon_3^{\rm Higgs}$	0 -4%	-7.7% $+3.7\%$	-10% $+6\%$
M_T^{eff} [GeV]	2×10^{14}	3×10^{18}	6×10^{19}

Goto & Nihei, Phys. Rev. D59: 115009,1999

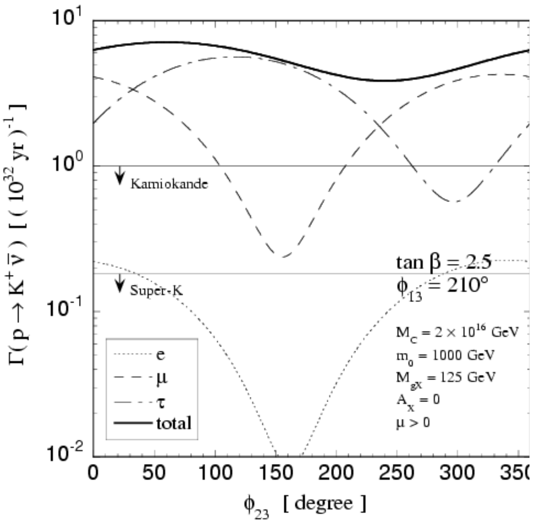


Figure 2: Decay rates $\Gamma(p \to K^+\overline{\nu}_i)$ ($i = e, \mu$ and τ) as functions of the phase ϕ_{23} for $\tan\beta = 2.5$. The other phase ϕ_{13} is fixed at 210°. The CKM phase is taken as $\delta_{13} = 90°$. We fix the soft SUSY breaking parameters as $m_0 = 1\,\mathrm{TeV}$, $M_{gX} = 125\,\mathrm{GeV}$ and $A_X = 0$. The sign of the supersymmetric Higgsino mass μ is taken to be positive. The colored Higgs mass M_C and the heavy gauge boson mass M_V are assumed as $M_C = M_V = 2 \times 10^{16}\,\mathrm{GeV}$. The horizontal lower line corresponds to the Super-Kamiokande limit $\tau(p \to K^+\overline{\nu}) > 5.5 \times 10^{32}$ years, and the horizontal upper line corresponds to the Kamiokande limit $\tau(p \to K^+\overline{\nu}) > 1.0 \times 10^{32}$ years.

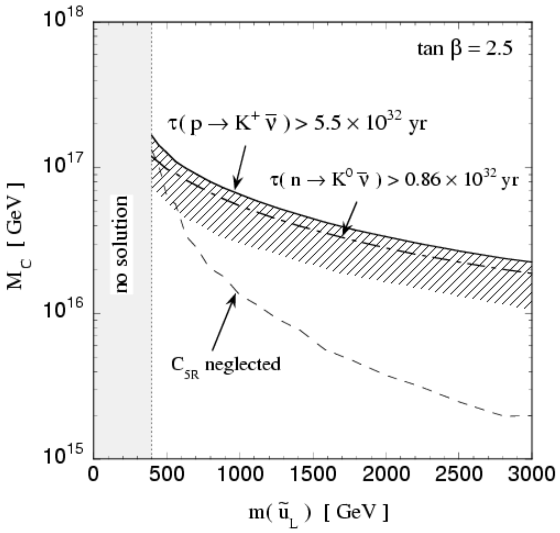


Figure 4: Lower bound on the colored Higgs mass M_C as a function of the left-handed scalar up-quark mass $m_{\tilde{u}_L}$. The soft breaking parameters m_0, M_{gX} and A_X are scanned within the range of $0 < m_0 < 3\,\mathrm{TeV}$, $0 < M_{gX} < 1\,\mathrm{TeV}$ and $-5 < A_X < 5$, and $\tan\beta$ is fixed at 2.5. Both signs of μ are considered. The whole parameter region of the two phases ϕ_{13} and ϕ_{23} is examined. The solid curve represents the bound derived from the Super-Kamiokande limit $\tau(p \to K^+\overline{\nu}) > 5.5 \times 10^{32}$ years, and the dashed curve represents the corresponding result without the $RRRR$ effect. Left-hand side of the vertical dotted line is excluded by other experimental constraints. The dash-dotted curve represents the bound derived from the Kamiokande limit on the neutron partial lifetime $\tau(n \to K^0\overline{\nu}) > 0.86 \times 10^{32}$ years.

Minimal SUSY SU(5) ruled out by proton decay + g.c. unification !!

Th: $\tau_{p \to K^+ \bar{\nu}} \leq (\frac{1}{3} - 3) \times 10^{34}$ yrs.

Ex: $\tau_{p \to K^+ \bar{\nu}} > 2.2 \times 10^{33}$ yrs. – Super-K

The natural value for ϵ_3 ?

- NO GUTs – ϵ_3 typically large due to large mass ratios for exotic states

$\epsilon_3 \sim$ O(100%)

- 4D SUSY GUTs – ϵ_3 protected by GUT and flavor symmetries

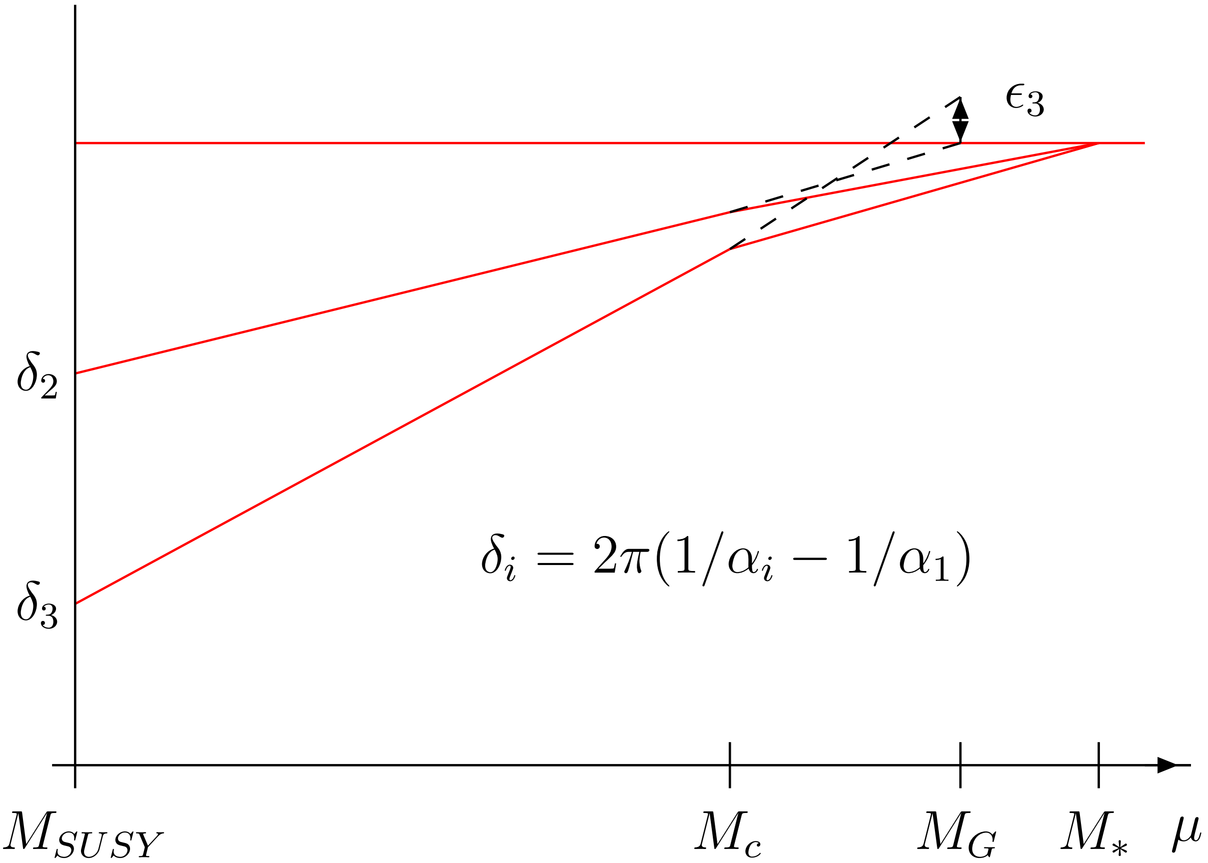
ϵ_3(GUT breaking sector) $\sim$ O(10%) possible

- 5D SUSY GUTs – ϵ_3 protected by GUT symmetry above M_c

ϵ_3(KK modes) – fixes compactification and cutoff scales

SUSY GUTs in 5 or 6 Dimensions

- G. Altarelli and F. Feruglio, “SU(5) grand unification in extra dimensions and proton decay,” [arXiv:hep-ph/0102301].
- L. J. Hall and Y. Nomura, “Gauge unification in higher dimensions,” [arXiv:hep-ph/0103125].
- Y. Nomura, D. R. Smith and N. Weiner, “GUT breaking on the brane,” [arXiv:hep-ph/0104041].
- T. Asaka, W. Buchmuller and L. Covi, “Gauge unification in six dimensions,” [arXiv:hep-ph/0108021].
- L. J. Hall, Y. Nomura, T. Okui and D. R. Smith, “SO(10) unified theories in six dimensions,” [arXiv:hep-ph/0108071].
- R. Contino, L. Pilo, R. Rattazzi and E. Trincherini, “Running and matching from 5 to 4 dimensions,” [arXiv:hep-ph/0108102].
- R. Dermisek and A. Mafi, “SO(10) grand unification in five dimensions: Proton decay and the mu problem,” [arXiv:hep-ph/0108139].
- A. Hebecker and J. March-Russell, “Proton decay signatures of orbifold GUTs,” [arXiv:hep-ph/0204037].
- H. D. Kim and S. Raby, “Unification in 5D SO(10),” [arXiv:hep-ph/0212348].


ϵ_3
δ_2
δ_3
$\delta_i = 2\pi(1/\alpha_i - 1/\alpha_1)$
M_{SUSY}
M_c
M_G
M_*
μ

GUT scale threshold corrections
Four vs. Five dimensions

- 4D

$\alpha, \quad \sin^2\theta_W, \quad \alpha_s$ fixes $\alpha_G, \quad M_G, \quad \epsilon_3(M_G)$

Minimal SU(5) : $\epsilon_3 = \frac{3\alpha_G}{5\pi}\ln(\frac{m_T}{M_G}) \sim -4\%$

$$\Longrightarrow M_G \simeq 3 \times 10^{16} \text{ GeV and}$$
$$m_T = 2 \times 10^{14} \text{ GeV}$$

- 5D

$\alpha, \quad \sin^2\theta_W, \quad \alpha_s$ fixes $\alpha_*, \quad M_*, \quad M_c$

$$\Longrightarrow M_* \simeq 10^{17} \text{ GeV and}$$
$$M_c \simeq 10^{14} \text{ GeV}$$

Novelty of 5D SUSY SO(10)

- Gauge symmetry breaking via Orbifold
- Higgs doublet–triplet splitting via Orbifold
- NO proton decay due to dim. 5 Operators $(p \longrightarrow K^{+}\ \bar{\nu})$

  $\Leftrightarrow$ R symmetry prevents dim. 5 ops.
- Proton decay due to dim. 6 operators $(p \longrightarrow e^{+}\ \pi^{0})$

  Negligible in 4D, however in 5D one is now sensitive to physics at the cutoff and the effects are incalculable (perhaps observable ?)

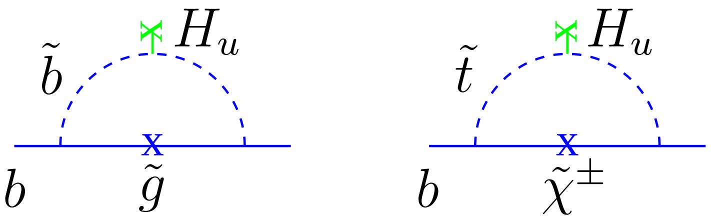
Yukawa unification

— Banks; Olechowski & Pokorski; Ananthanarayan, Lazarides & Shafi; Dimopoulos, Hall & SR, Baer & Ferrandis and T. Blažek, R. Dermíšek & SR

- Insignificant GUT threshold corrections from gauge and Higgs loop
- Weak scale threshold corrections $\propto \tan\beta \sim m_t/m_b$

— Hall, Rattazzi, & Sarid; Carena, Olechowski, Pokorski & Wagner; Blažek, Pokorski & SR; Pierce, Bagger, Matchev & Zhang

- $\delta m_b/m_b = \Delta m_b^{\tilde{g}} + \Delta m_b^{\tilde{\chi}} + \Delta m_b^{Log} + \cdots$
- $\Delta m_b^{\tilde{g}} \sim -\Delta m_b^{\tilde{\chi}} > 0$ [Can be ~ 50 %]



- $\Delta m_b^{Log} \sim +6$ %

Good fits require $\delta m_b/m_b < -2\%$

Bottom Line

Yukawa unification possible only in a narrow region of SUSY parameter space

- T. Blazek, R. Dermisek and S. Raby, "Predictions for Higgs and SUSY spectra from SO(10) Yukawa unification with $\mu > 0$," [arXiv:hep-ph/0107097].
- T. Blazek, R. Dermisek and S. Raby, "Yukawa unification in SO(10)," [arXiv:hep-ph/0201081].
- K. Tobe and J. D. Wells, "Revisiting top-bottom-tau Yukawa unification in supersymmetric grand unified theories," [arXiv:hep-ph/0301015].
- D. Auto, H. Baer, C. Balazs, A. Belyaev, J. Ferrandis and X. Tata, "Yukawa coupling unification in supersymmetric models," [arXiv:hep-ph/0302155].
- R. Dermisek, S. Raby, L. Roszkowski and R. Ruiz De Austri, "Dark matter and $B_s \to \mu^+\mu^-$ with minimal SO(10) soft SUSY breaking," [arXiv:hep-ph/0304101].

Minimal SO(10) SUSY Model

Gauge coupling unification

$\alpha_G, \; M_G, \; \epsilon_3 \sim -4\%$

Yukawa unification

$\lambda_t \; = \; \lambda_b \; = \; \lambda_\tau \; = \; \lambda_{\bar{\nu}_\tau} \; \equiv \; \lambda$

Soft SUSY breaking

$m_{16}, \; m_{10}, \; A_0, \; M_{1/2}, \; \tan\beta, \; \Delta m_H^2$

$A_0 \sim -2\, m_{16}$

$m_{10} \sim \sqrt{2}\, m_{16}$

$m_{16} \geq 2 \text{ TeV} \gg \mu, \; M_{1/2}$

$\Delta m_H^2 \sim 10\%$

$\tan\beta \approx 50$

“Natural” Inverted Scalar Mass Hierarchy

Heavy 1st & 2nd generation squarks and sleptons >> TeV ; Light 3rd generation scalars $\leq$ TeV

J. A. Bagger, J. L. Feng, N. Polonsky and R. J. Zhang, “Superheavy supersymmetry from scalar mass A-parameter fixed points,” [arXiv:hep-ph/9911255].

Analyze RG running of soft SUSY breaking in MSSM*

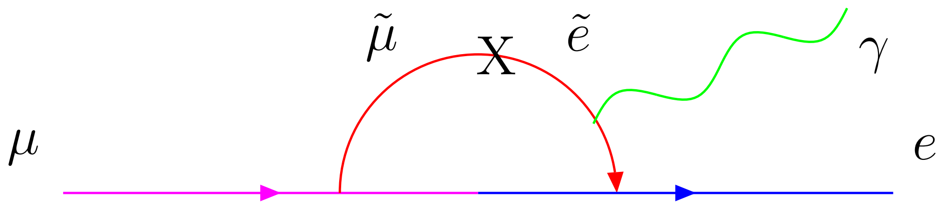
Find initial conditions such that

$$S = \frac{\tilde{m}^2_{1,2}}{\tilde{m}^2_3} \gg 1$$

Same region of soft SUSY breaking parameter space !!

Suppress flavor & CP violation

- Example : $\mu \to e\ \gamma$



- SUSY flavor basis = lepton mass eigenbasis, BUT scalar masses are not necessarily diagonal in same basis !

$$\delta^l_{12} = \delta m^2_{e\mu} / \tilde{m}^2$$

$$BR(\mu \to e\ \gamma) \propto (\delta^l_{12}/\tilde{m}^2)^2$$

- $B(\mu \to e\gamma) < 1.2 \times 10^{-11} \Longrightarrow$

$|(\delta^l_{12})_{LL}| < 2.1 \times 10^{-3} (m_{\tilde{l}}(\mathrm{GeV})/100)^2$

$$\boxed{|(\delta^l_{12})_{LL}| < 0.8\ (m_{\tilde{l}}(\mathrm{TeV})/2)^2}$$

Phenomenology of MSO$_{10}$SM

- Higgs mass

$\tan\beta \sim 50$
light stop, sbottom, stau

$$m_h = 114 \pm 5 \pm 3 \text{ GeV}$$

Figure

- Anomalous magnetic moment of muon

$m_{16} \geq 2$ TeV

$$a_\mu^{SUSY} \leq 6 \times 10^{-10}$$

- $\tilde{\chi}^0$ LSP – Dark Matter

Dermíšek, SR, Roszkowski & Ruiz de Austri, JHEP **0304**, 037 (2003)

Large $\tan\beta \Longrightarrow (\tilde{\chi}^0\tilde{\chi}^0 \rightarrow A^0 \rightarrow \text{hadrons})$
enhanced

Figure

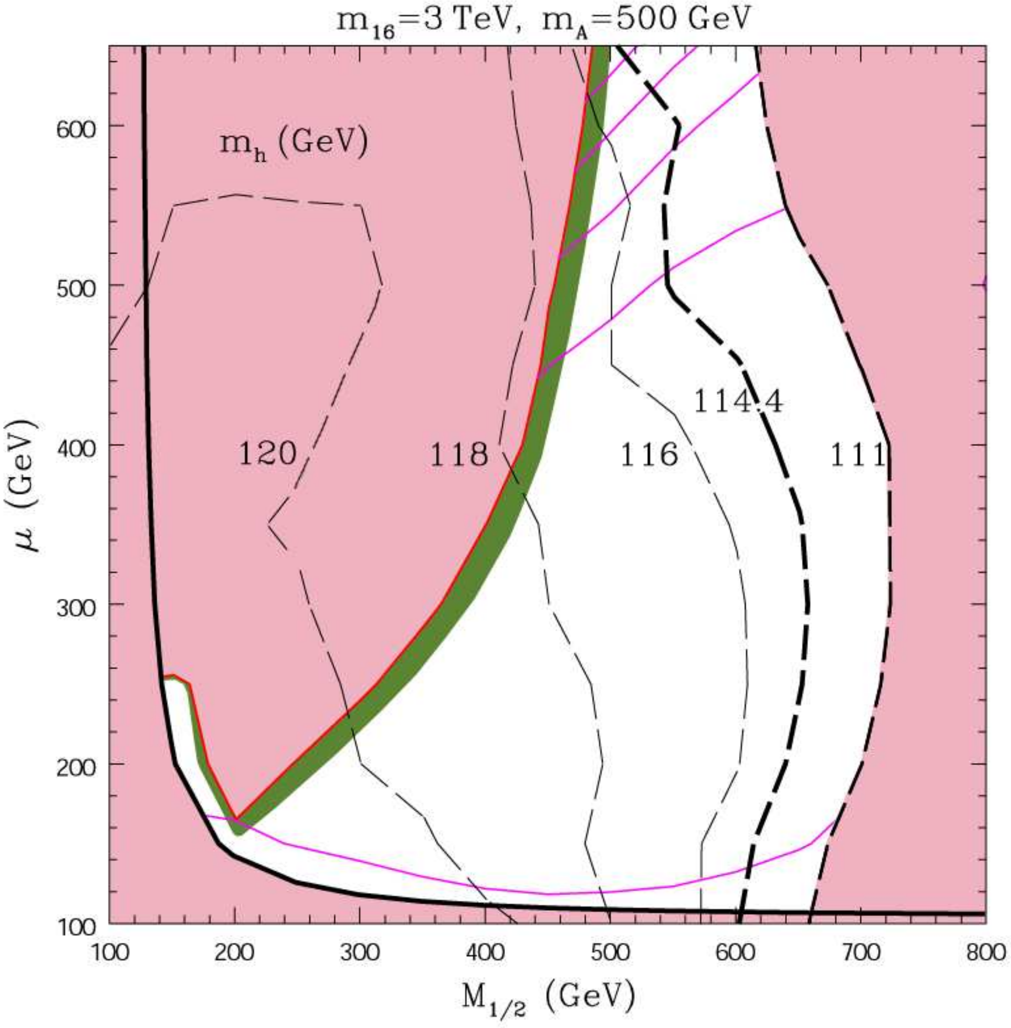
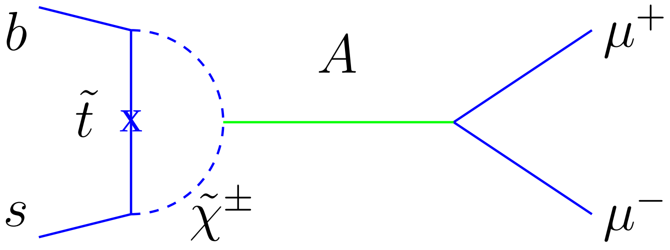


Figure 1: Dermíšek, SR, Roszkowski & Ruiz de Austri

Large $\tan\beta \sim 50$ and Quark Flavor Violation

- Huang C S, Liao W and Yan Q S "The promising process to distinguish supersymmetric models with large tan(beta) from the standard model: $B \to X_s \mu^+ \mu^-$," [arXiv:hep-ph/9803460].
- C. Hamzaoui, M. Pospelov and M. Toharia, "Higgs-mediated FCNC in supersymmetric models with large tan(beta)," [arXiv:hep-ph/9807350].
- K. S. Babu and C. F. Kolda, "Higgs-mediated $B_0 \to \mu^+ \mu^-$ in minimal supersymmetry," [arXiv:hep-ph/9909476].
- Chankowski P H and Slawianowska L "$B_{0d,s} \to \mu^- \mu^+$ decay in the MSSM," [arXiv:hep-ph/0008046].
- A. Dedes, H. K. Dreiner and U. Nierste, "Correlation of $B_s \to \mu^+ \mu^-$ and $(g-2)_\mu$ in minimal supergravity," [arXiv:hep-ph/0108037].
- G. Isidori and A. Retico, "Scalar flavour-changing neutral currents in the large-tan(beta) limit," [arXiv:hep-ph/0110121].

$$B(B_s \to \mu^+\mu^-) \propto \tan\beta^4$$



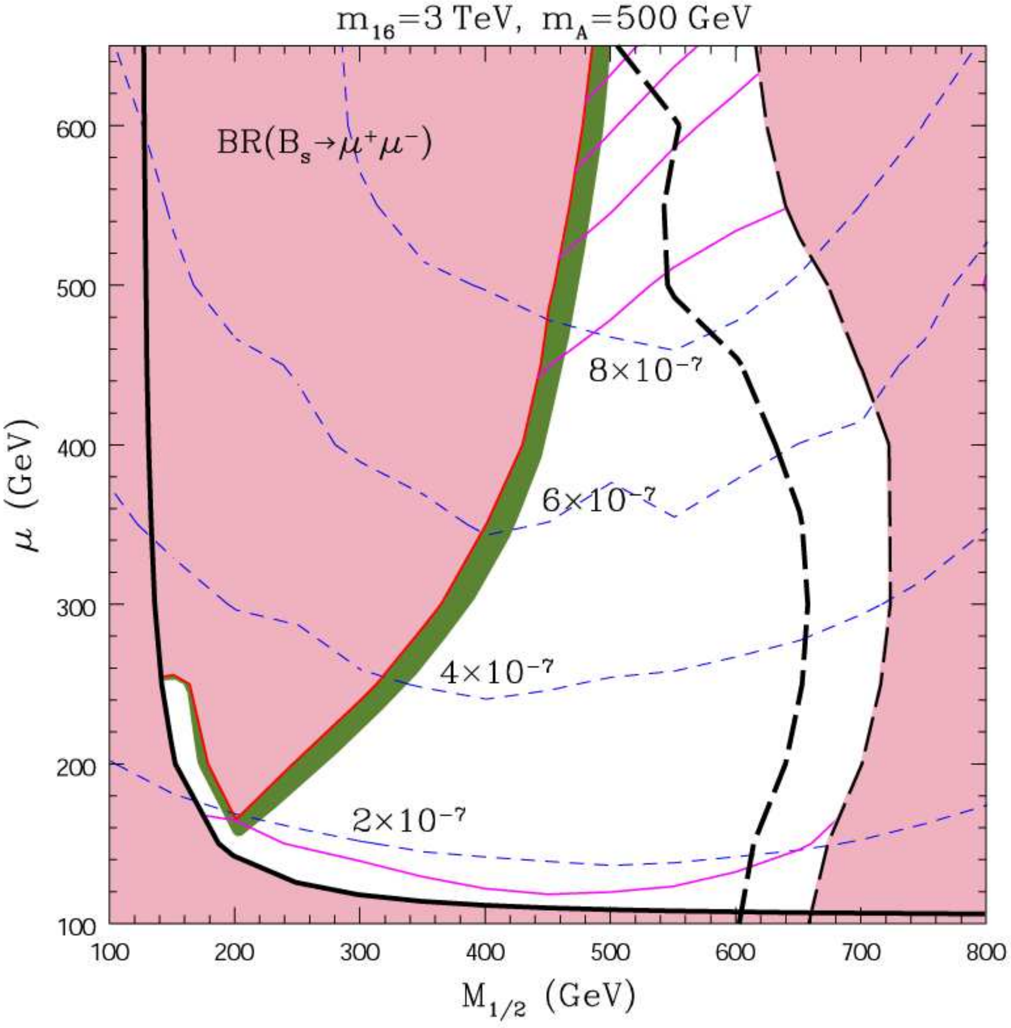


Figure 2: Dermíšek, SR, Roszkowski & Ruiz de Austri

SUSY GUTs protect against large corrections to perturbative unification

Non-abelian Family Symmetries protect against large flavor violation and can explain the hierarchy of quark and lepton masses and mixing angles

Patterns of Masses and Mixing

- $\lambda_t = \lambda_b = \lambda_\tau = \lambda_{\nu_\tau}$ $\qquad SO_{10}@M_G$

- $\lambda_s \sim \frac{1}{3}\lambda_\mu, \quad \lambda_d \sim 3\lambda_e$ $\qquad @M_G$

— Georgi & Jarlskog; Harvey, Ramond & Reiss

$$\Rightarrow m_s \approx 4 \cdot \tfrac{1}{3} m_\mu, \quad m_d \approx 4 \cdot 3 m_e \qquad @M_Z$$

- $\lambda_d \lambda_s \lambda_b \approx \lambda_e \lambda_\mu \lambda_\tau$ $\qquad SU_5@M_G$

$$\Rightarrow Det(m_d) \approx Det(m_e) \qquad @M_G$$

- $V_{us} \approx \sqrt{m_d/m_s}$

— Fritzsch; Weinberg; Wilczek & Zee

- $V_{ub}/V_{cb} \approx \sqrt{m_u/m_c}$

- $V_{cb} \sim m_s/m_b \sim \sqrt{m_c/m_t}$

— Harvey, Ramond & Reiss

Family Symmetry

- Family Hierarchy U_1

— Froggatt & Nielsen; Dimopoulos et al.; Ramond et al.; Seiberg et al.

example

$$m_d = \begin{pmatrix} 0 & \epsilon' & \epsilon' \\ \epsilon' & \epsilon & \epsilon \\ 0 & 1 & 1 \end{pmatrix} m_b$$

where

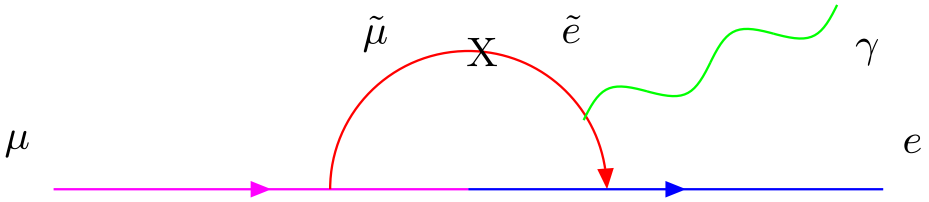
$$\epsilon = \frac{\langle \Theta^- \rangle}{M}, \quad \epsilon' = \frac{\langle \Theta^+ \rangle}{M}$$

$$V_{cb} \approx m_s/m_b \approx \epsilon$$

$$V_{us} \approx \sqrt{m_d/m_s} \approx \epsilon'/\epsilon$$

- SU_2 family symmetry — Flavor Problem

— Dine, Kagan & Leigh; Pomarol & Tomassini; Barbieri, Dvali & Hall



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$SO_{10} \times [SU_2 \times U_1^n]_{FS}$

— Barbieri, Hall, S.R. & Romanino; Blazek, S.R. & Tobe

The superpotential for the charged fermion sector, including the heavy Froggatt-Nielsen states — $\{ \chi \}$

$$\begin{aligned} W \supset \quad & 16_3 \; 10 \; 16_3 + 16_a \; 10 \; \chi^a \\ & + \bar{\chi}_a \left(M_\chi \; \chi^a + \; 45 \; \frac{\phi^a}{\hat{M}} \; 16_3 \; + \; 45 \; \frac{S^{ab}}{\hat{M}} \; 16_b + A^{ab} \; 16_b \right) \end{aligned}$$

where

- $\begin{matrix} 16_a & a, b = 1, 2 \\ & 16_3 \end{matrix}$ — 3 families
- $10 \Longrightarrow$ Higgs doublets
- $M_\chi = M_0 (1 \; + \; \alpha \, X \; + \; \beta \, Y)$

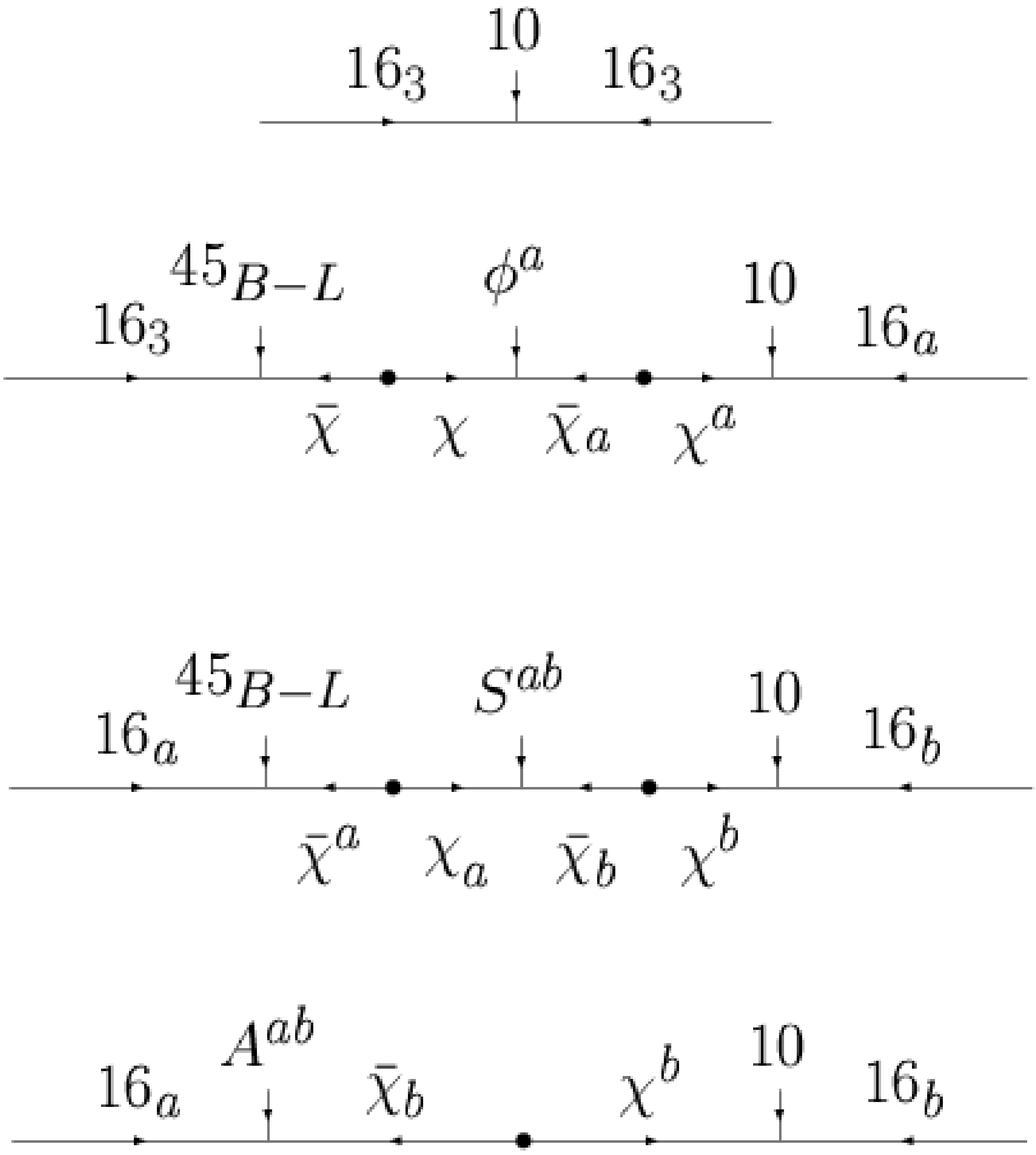
X, Y — SO(10) breaking vevs in the adjoint representation

X corresponding to the U(1) in SO(10) which preserves SU(5),

Y is standard weak hypercharge and

- α, β are arbitrary parameters.

Effective Fermion Mass Operators



Effective Yukawa Couplings

$$Y_u = \begin{pmatrix} 0 & \epsilon' \, \rho & -\epsilon \, \xi \\ -\epsilon' \, \rho & \tilde{\epsilon} \, \rho & -\epsilon \\ \epsilon \, \xi & \epsilon & 1 \end{pmatrix} \lambda$$

$$Y_d = \begin{pmatrix} 0 & \epsilon' & -\epsilon \, \xi \, \sigma \\ -\epsilon' & \tilde{\epsilon} & -\epsilon \, \sigma \\ \epsilon \, \xi & \epsilon & 1 \end{pmatrix} \lambda \qquad (1)$$

$$Y_e = \begin{pmatrix} 0 & -\epsilon' & 3 \, \epsilon \, \xi \\ \epsilon' & 3 \, \tilde{\epsilon} & 3 \, \epsilon \\ -3 \, \epsilon \, \xi \, \sigma & -3 \, \epsilon \, \sigma & 1 \end{pmatrix} \lambda$$

with

$$\begin{aligned} \xi &= \phi^1/\phi^2; & \tilde{\epsilon} &\propto (S^{22}/\hat{M}); \qquad (2) \\ \epsilon &\propto \phi^2/\hat{M}; & \epsilon' &\sim (A^{12}/M_0); \\ \sigma &= \frac{1+\alpha}{1-3\alpha}; & \rho &\sim \beta \ll \alpha. \end{aligned}$$

Features of the Model

1. Family Hierarchy

$$SU_2 \times U_1 \quad \underset{\epsilon}{\longrightarrow} \quad U_1 \quad \underset{\epsilon'}{\longrightarrow} \quad \text{nothing}$$

3rd family >> 2nd family >> 1st family

2. Patterns - approximate Georgi - Jarlskog "natural"

$$\langle 45 \rangle = (B - L) M_G$$

$$m_s \sim \tfrac{1}{3} m_\mu$$

$$m_d \sim 3 m_e$$

3. $\lambda_t = \lambda_b = \lambda_\tau = \lambda_{\nu_\tau} = \lambda \; @M_G$
4. $m_u < m_d$ even though $m_t >> m_b$
5. Gauge Coupling Unification
6. SU_2 suppresses flavor violation such as $\mu \to e\gamma$
7. 10 Yukawa parameters fit 13 fermion masses and mixing angles

Observable	Data(σ) (masses	*Theory* in GeV)
M_Z	91.187 (0.091)	91.18
M_W	80.388 (0.080)	80.40
$G_\mu \cdot 10^5$	1.1664 (0.0012)	1.166
α_{EM}^{-1}	137.04 (0.14)	137.0
$\alpha_s(M_Z)$	0.1190 (0.003)	0.1174
$\rho_{new} \cdot 10^3$	-0.20 (1.1)	$+0.322$
M_t	173.8 (5.0)	175.0
$m_b(M_b)$	4.260 (0.11)	4.326
$M_b - M_c$	3.400 (0.2)	3.432
m_s	0.180 (0.050)	0.146
m_d/m_s	0.050 (0.015)	0.0585
Q^{-2}	0.00203 (0.00020)	0.00201
M_τ	1.777 (0.0018)	1.776
M_μ	0.10566 (0.00011)	.1057
$M_e \cdot 10^3$	0.5110 (0.00051)	0.5110
V_{us}	0.2205 (0.0026)	0.2206
V_{cb}	0.03920 (0.0030)	0.0402
V_{ub}/V_{cb}	0.0800 (0.02)	0.0702
$\hat{B}_K$	0.860 (0.08)	0.8691
$B(b \rightarrow s\gamma) \cdot 10^4$	3.000 (0.47)	2.958
TOTAL χ^2	2.48	

Neutrinos : Masses and Mixing Angles

- $\Delta m^2_{atm} = |m_3^2 - m_2^2| \approx 3 \times 10^{-3}\ \mathrm{eV}^2$
  $\sin 2\theta_{atm} \approx 1$
- $\Delta m^2_{sol} = |m_2^2 - m_1^2| \approx 7 \times 10^{-5}\ \mathrm{eV}^2$
  $\sin 2\theta_{sol} \leq 1$

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} \approx \begin{pmatrix} c_{sol} & s_{sol} & 0 \\ -s_{sol}/\sqrt{2} & c_{sol}/\sqrt{2} & 1/\sqrt{2} \\ -s_{sol}/\sqrt{2} & c_{sol}/\sqrt{2} & -1/\sqrt{2} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

Neutrinos masses $\sim m_D^T\ M_N^{-1}\ m_D$

Bi-large Neutrino Mixing in $SO_{10} \times [SU_2 \times U_1^n]_{FS}$ model

$$Y_\nu = \begin{pmatrix} 0 & -\epsilon' \; \omega & \frac{3}{2} \; \epsilon \; \xi \; \omega \\ \epsilon' \; \omega & 3 \; \tilde{\epsilon} \; \omega & \frac{3}{2} \; \epsilon \; \omega \\ -3 \; \epsilon \; \xi \; \sigma & -3 \; \epsilon \; \sigma & 1 \end{pmatrix} \lambda$$

with $\omega \;\; = \;\; 2\,\sigma/(2\,\sigma - 1)$

$\Longrightarrow$ Dirac neutrino mass matrix – $m_\nu \equiv Y_\nu \frac{v}{\sqrt{2}} \sin\beta$

$$W_{neutrino} = \;\; \frac{\overline{16}}{\hat{M}} \left(N_1 \; \tilde{\phi}^a \; 16_a \; + \; N_2 \; \phi^a \; 16_a \; + \; N_3 \; \theta \; 16_3\right) + \frac{1}{2} \left(S_1 \; N_1^2 \; + \; S_2 \; N_2^2\right)$$

$$S^{ab} \equiv \tilde{\phi}^a \; \tilde{\phi}^b / \hat{M}$$

- $\mathcal{M} = U_e^{tr} \; [\; D^{tr} \; \hat{M}_N^{-1} \; D \;] \; U_e$

with $D^{tr} = \begin{pmatrix} a & 0 \\ a' & b \\ 0 & b' \end{pmatrix}$ and $\hat{M}_N \equiv \begin{pmatrix} \langle S_1 \rangle & 0 \\ 0 & \langle S_2 \rangle \end{pmatrix}$

- $b \sim b'$ and $a \sim a' \ll b$

Gives “natural” Glashow, Frampton & Yanagida ansatz

$\Longrightarrow$ Bi-large neutrino mixing matrix

Conclusions

- SUSY GUTs – “Natural” extension of Standard Model
- Bounty of NEW Particles at the LHC
- Decades of EXCITEMENT disentangling the new phenomena
- Opening a Window into the GUT/Planck scale

T minus 3 years
and COUNTING !